LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2014**

ST 1820/1815 - ADVANCED DISTRIBUTION THEORY

Date : 29/03/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Let X be the number of heads obtained when a coin is tossed twice. Verify whether X is a random variable or not.
2. Mention the pdf of truncated binomial, left truncated at ‘0’ and obtain its mgf.
3. Verify whether or not the truncated Poisson distribution, truncated at zero, is a power series distribution.
4. Check that geometric distribution satisfies lack of memory property.
5. Let X be distributed as Lognormal. Find the distribution of $\frac{1}{X}$.
6. Prove that 2X is Inverse Gaussian, when X is Inverse Gaussian.
7. Derive the pgf of power-series distribution and hence find its mgf.
8. Find the marginal distributions of X_1 and X_2, when $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$.
9. Establish additive property of bivariate Poisson distribution.
10. If $X \sim B(2, \theta)$, $\theta = 0.1, 0.2, 0.3$ and if θ is discrete uniform, then find the mean of the compound distribution.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Let the distribution function of a random variable X be

$$F(x) = \begin{cases} 0, & x < 2 \\ \left(\frac{2}{3}\right)x - 1, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$$

    Obtain (i) the decomposition of F,
    (ii) mgf of X.
12. Obtain a characterization of Poisson distribution through pdf.
13. Derive the mgf of inverse Gaussian distribution.
14. Stating the conditions, prove that $BB(n, p_1, p_2, p_{12})$ tends to $BVP(\lambda_1, \lambda_2, \lambda_{12})$.
15. Obtain the regression equations associated with bivariate Poisson distribution.
16. State Skitovitch theorem for normal distributions and give its proof.
17. Derive the mean and variance of non-central F distribution.
18. Given a random sample from a normal distribution, using the theory of quadratic forms, check whether or not the sample mean is independent of the sample variance.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a) Derive the recurrence formula for finding r^{th} cumulant k_r for a power-series distribution. Hence obtain k_r for Poisson distribution. (10)

(b) Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Prove that $X_1 \mid X_2 = x_2 \overset{d}{=} U_1 + V_1$, where $U_1 \sim B(n - x_2, \frac{p_1}{q + p_1})$, $V_1 \sim B(x_2, \frac{p_{12}}{p_2 + p_{12}})$ and U_1 is independent of V_1. (10)

20(a) Show that mean > median > mode for a log-normal distribution. (10)

(b) Find the conditional distribution of (i) $X_2 \mid X_1 = x_1$ and (ii) $X_1 \mid X_2 = x_2$, when $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (10)

21(a) Show that $(X - \mu)^2 / (\mu^2 X) \sim \chi^2(1)$, when $X \sim IG(\mu, \lambda)$. (10)

(b) Given that X_1 and X_2 are two independent normal variables with the same variance. State and prove a necessary and sufficient condition for two linear combinations of X_1 and X_2 to be independent. (10)

22(a) Derive the mgf of (X_1, X_2) at (t_1, t_2), when $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (10)

(b) Derive the pdf of non-central t – distribution. (10)

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